High-Accuracy Preintegration for Visual-Inertial Navigation

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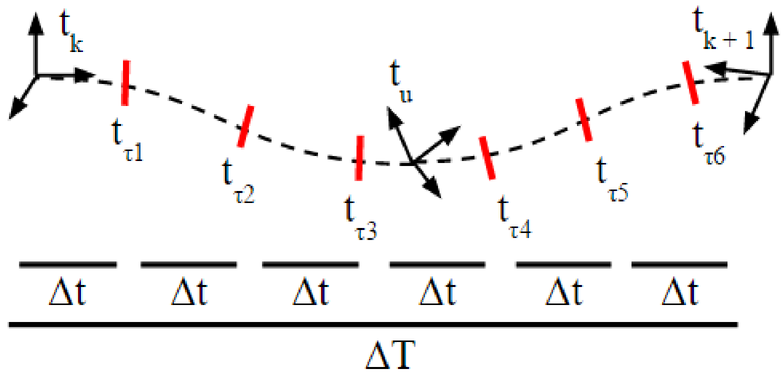
Motivation

- Inertial Measurement Unit (IMU):
  - Pros:
    - Provide a high frequency of acceleration (accelerometer) and angular velocity (gyro) data
    - Have become cheap and lightweight in recent years
  - Cons:
    - MEMS-IMU measurements are noisy and corrupted by time-varying biases
    - Hard to optimally fuse with other sensors (cameras) at IMU-rates

Preintegration

- Preintegration: Integrate multiple IMU measurements in local frame of reference [Lupton et al.'12]



Position: $${}^G\mathbf{p}_{k+1} = {}^G\mathbf{p}_k + {}^G\mathbf{v}_k \Delta T - \frac{1}{2}{}^G\mathbf{g}\Delta T^2 + {}^G_k\mathbf{R}\underbrace{\int_{t_k}^{t_{k+1}}\int_{t_k}^{s} {}^k_u\mathbf{R}\left({}^u\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du ds}_{{}^k\boldsymbol{\alpha}_{k+1}}$$

Velocity: $${}^G\mathbf{v}_{k+1} = {}^G\mathbf{v}_k - {}^G\mathbf{g}\Delta T + {}^G_k\mathbf{R}\underbrace{\int_{t_k}^{t_{k+1}} {}^k_u\mathbf{R}\left({}^u\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du}_{{}^k\boldsymbol{\beta}_{k+1}}$$

Rotation: $${}^{k+1}_G\mathbf{R} = {}^{k+1}_k\mathbf{R}\,{}^k_G\mathbf{R}$$

Preintegration (cont.)

- Arrive at three preintegrated measurements

$$ {}^{k}_{G}\mathbf{R}\left({}^{G}\mathbf{p}_{k+1} - {}^{G}\mathbf{p}_{k} - {}^{G}\mathbf{v}_{k}\Delta T + \frac{1}{2}{}^{G}\mathbf{g}\Delta T^{2}\right) = {}^{k}\boldsymbol{\alpha}_{k+1} $$

$$ {}^{k}_{G}\mathbf{R}\left({}^{G}\mathbf{v}_{k+1} - {}^{G}\mathbf{v}_{k} + {}^{G}\mathbf{g}\Delta T\right) = {}^{k}\boldsymbol{\beta}_{k+1} $$

$$ {}^{k+1}_{G}\mathbf{R}\,{}^{k}_{G}\mathbf{R}^{\top} = {}^{k+1}_{k}\mathbf{R} $$

- Can be used in estimation techniques such as batch optimization

- Need to compute measurement ***mean*** and ***covariance***

Preintegration: Related Work

- State-of-the-art approach [Forster et al. ‘15]:
  - Pro: Stable Lie algebra representation of SO(3)
  - Con: Based on ***discrete*** measurement dynamics

- The proposed approach:
  - Formulate and solve preintegration in ***continuous*** time to better model the underlying dynamics
  - Derive ***closed-form*** expressions for preintegration measurements, covariance, and bias Jacobians

Preintegrated Measurement Mean

- Relative rotation, ${}^{k+1}_{k}\mathbf{R}$, computed with quaternion integration

$$ {}^{u}_{k}\dot{\bar{q}} = \frac{1}{2}\Omega\left(\omega_m - \mathbf{b}_w - \mathbf{n}_w\right){}^{u}_{k}\bar{q} \;\Rightarrow\; {}^{u}_{k}\dot{\hat{\bar{q}}} = \frac{1}{2}\Omega(\omega_m - \bar{\mathbf{b}}){}^{u}_{k}\hat{\bar{q}} \qquad \boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -\lfloor \boldsymbol{\omega} \times \rfloor & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^\top & 0 \end{bmatrix} $$

- Preintegrated measurements, ${}^{k}\alpha_{k+1}$ and ${}^{k}\beta_{k+1}$, formulated as linear system

$$ \underbrace{\int_{t_k}^{t_{k+1}} \int_{t_k}^{s} {}^{k}_{u}\mathbf{R}\left({}^{u}\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du ds}_{{}^{k}\boldsymbol{\alpha}_{k+1}} $$

$$ \underbrace{\int_{t_k}^{t_{k+1}} {}^{k}_{u}\mathbf{R}\left({}^{u}\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du}_{{}^{k}\boldsymbol{\beta}_{k+1}} $$

$$ \Rightarrow \begin{bmatrix} {}^{k}\dot{\hat{\boldsymbol{\alpha}}}_u \\ {}^{k}\dot{\hat{\boldsymbol{\beta}}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{3\times 3} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} {}^{k}\hat{\boldsymbol{\alpha}}_u \\ {}^{k}\hat{\boldsymbol{\beta}}_u \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ {}^{k}_{u}\hat{\mathbf{R}} \end{bmatrix} \left({}^{u}\mathbf{a}_m - \bar{\mathbf{b}}_a\right) $$

- **Closed form** solutions for all preintegrated measurements

Bias-Independent Preintegration

- Evaluating measurement means involves solving a nonlinear function wrt. biases:

$$ {}^{k}\hat{\alpha}_{k+1} = f(\mathcal{I}, \mathbf{b}) $$

  - Values depend on current linearization point for bias

- Bias Jacobians: remove preintegration dependency on biases via first-order Taylor-series expansions :

$$ {}^{k}\hat{\alpha}_{k+1} \approx f(\mathcal{I}, \bar{\mathbf{b}}) + \left.\frac{\partial f}{\partial \mathbf{b}}\right|_{\bar{\mathbf{b}}} (\mathbf{b} - \bar{\mathbf{b}}) $$

  - Allows for *efficient* measurement corrections due to bias changes

Preintegrated Measurement Covariance

- Linear system derived for error state

$$\begin{bmatrix} {}^k\delta\dot{\boldsymbol{\alpha}}_u \\ {}^k\delta\dot{\boldsymbol{\beta}}_u \\ {}^u\delta\dot{\boldsymbol{\theta}}_k \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I}_{3\times 3} & 0 \\ 0 & 0 & -{}^k_u\hat{\mathbf{R}}\left\lfloor \left({}^u\mathbf{a}_m - \bar{\mathbf{b}}_a\right)\times\right\rfloor \\ 0 & 0 & -\left\lfloor\left({}^u\boldsymbol{\omega}_m - \bar{\mathbf{b}}_w\right)\times\right\rfloor \end{bmatrix} \begin{bmatrix} {}^k\delta\boldsymbol{\alpha}_u \\ {}^k\delta\boldsymbol{\beta}_u \\ {}^u\delta\boldsymbol{\theta}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -{}^k_u\hat{\mathbf{R}} & 0 \\ 0 & -\mathbf{I}_{3\times 3} \end{bmatrix} \begin{bmatrix} \mathbf{n}_a \\ \mathbf{n}_w \end{bmatrix}$$

$$\Rightarrow \quad \dot{\mathbf{r}} = \mathbf{F}\mathbf{r} + \mathbf{G}\mathbf{n}$$

- *Closed form* discrete-time state-transition matrix

$$\dot{\boldsymbol{\Phi}}(t, t_0) = \mathbf{F}(t)\boldsymbol{\Phi}(t, t_0)$$

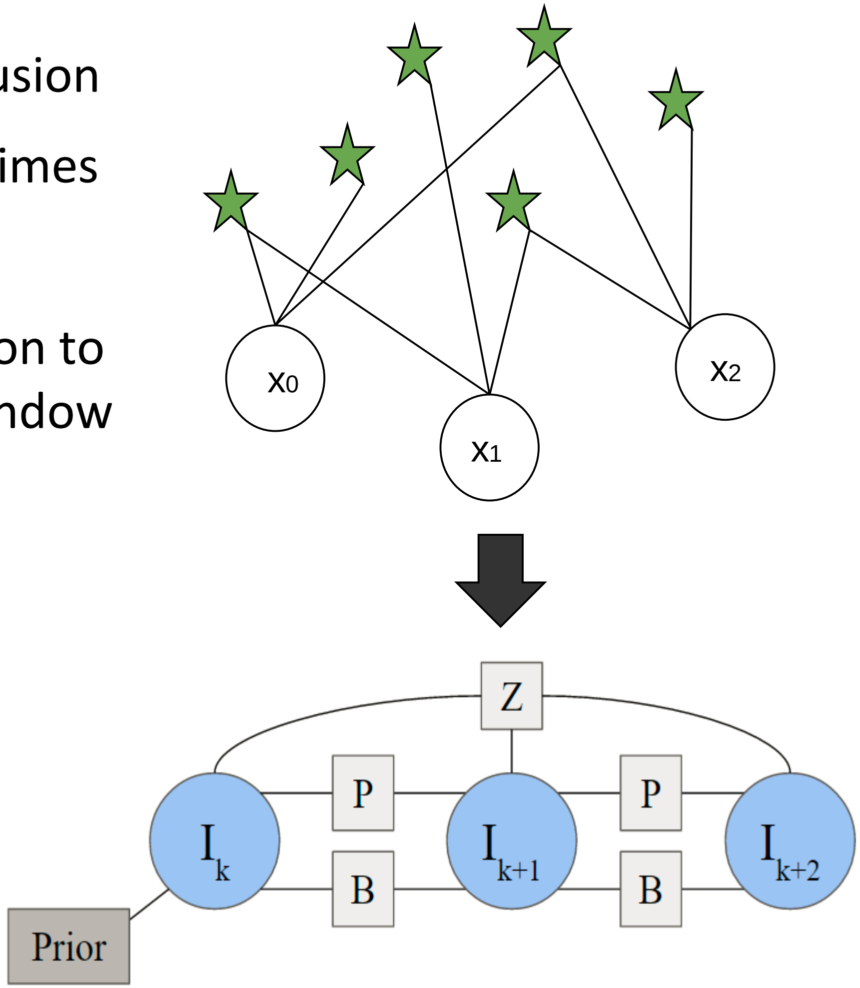
$$\boldsymbol{\Phi}\left(t_0, t_0\right) = \mathbf{I}$$

- Measurement noise covariance computed iteratively

$$\mathbf{P}_{\tau+1} = \boldsymbol{\Phi}(t_{\tau+1}, t_\tau)\mathbf{P}_\tau\boldsymbol{\Phi}(t_{\tau+1}, t_\tau)^\top + \mathbf{Q}_d$$

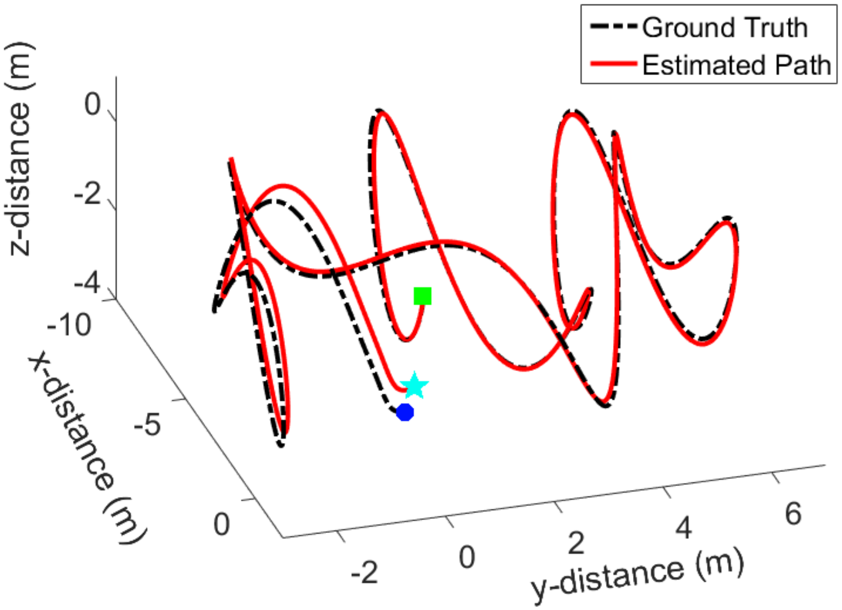
Graph-based Visual-Inertial Navigation

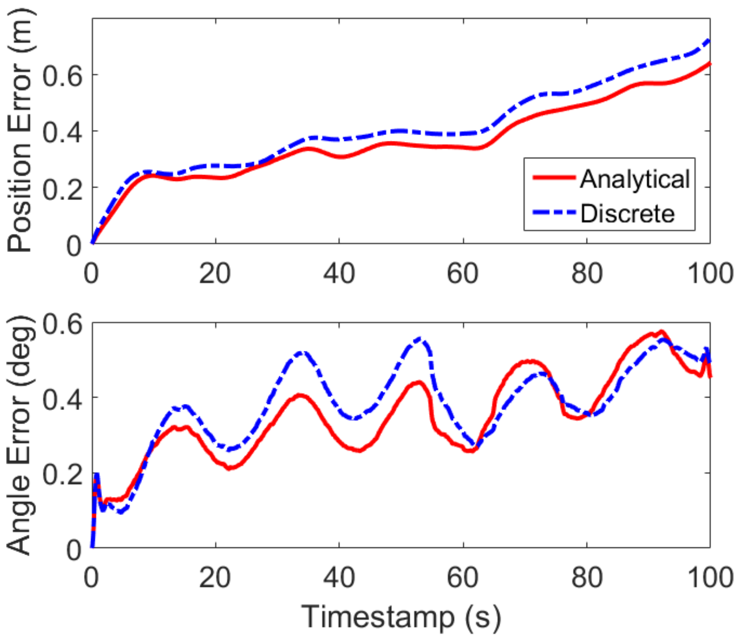
- Loosely coupled visual-inertial sensor fusion
- Store poses corresponding to imaging times
- Visual factors:
  - Use local, relative batch optimization to find distribution across a sliding window of states and features detected
  - Marginalize out features to yield constraint of only states
- Fuse w. preintegration and bias factors



Simulation Results

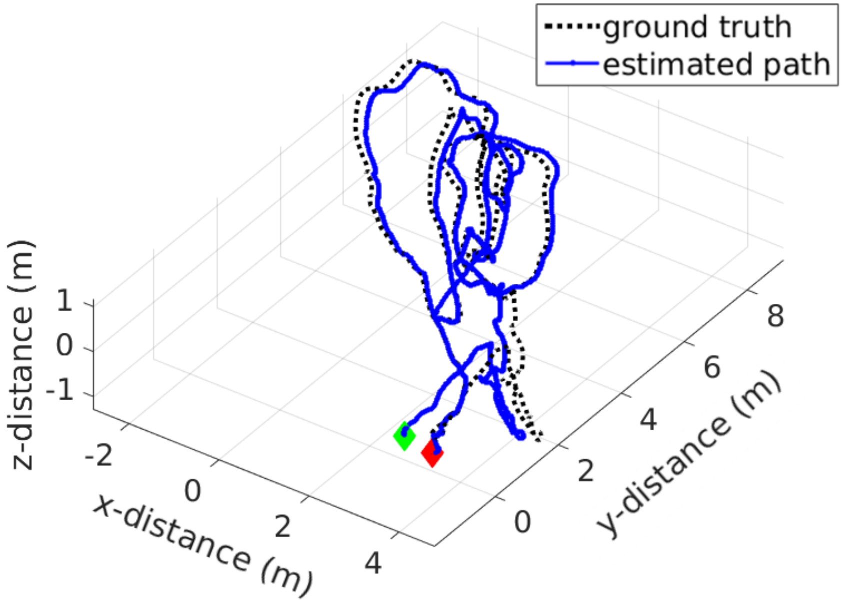
- Monte Carlo simulations with highly-dynamic motion
  - Our proposed approach (continuous) vs. state-of-the-art approach (discrete) [Forster et al. '15]

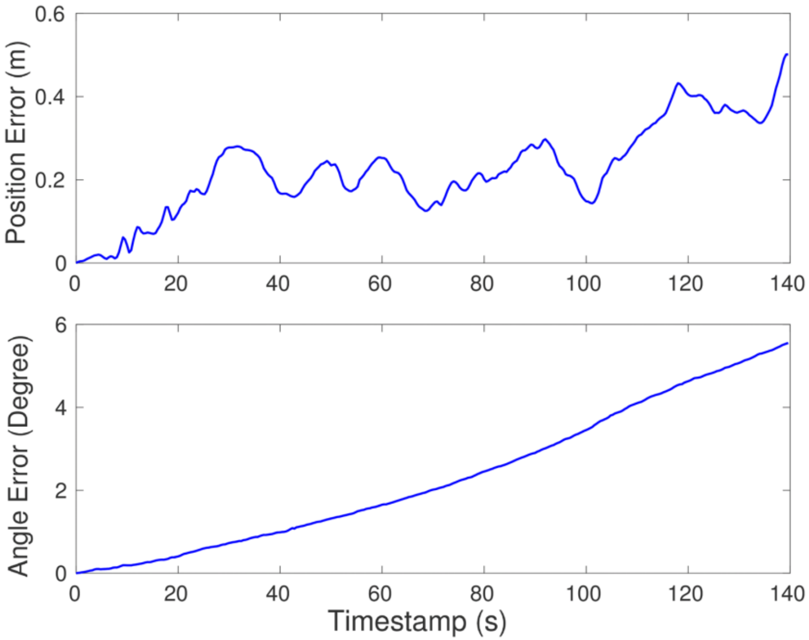




Experimental Results

- Full system validated on The EuRoC MAV Dataset [Burri '16]
- Achieved 0.7% translational drift





Summary

- Formulated and ***analytically*** solved preintegration in ***continuous*** time
- Shown to outperform a state-of-the art counterpart (in particular, in the case of highly-dynamic motion)
- Extensions (Dirty Laundry):
  - To investigate effect of higher IMU measurement rates
  - To investigate higher order modeling (spline) of measurements to solve discrete sampling problem
  - To integrate into various aided inertial navigation systems

Thank you!