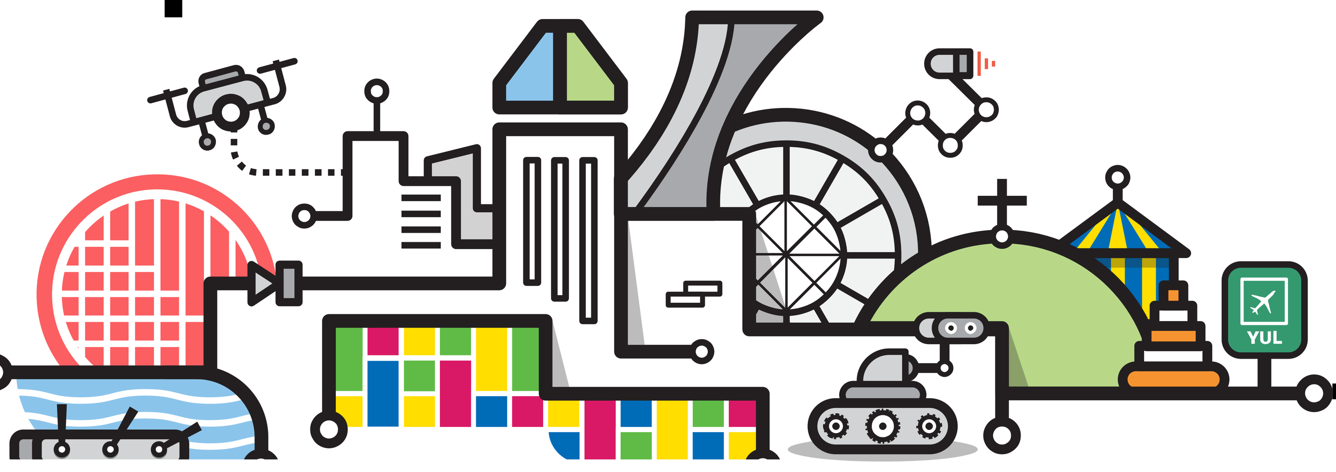
Degenerate Motion Analysis for Aided INS with Online Spatial and Temporal Sensor Calibration

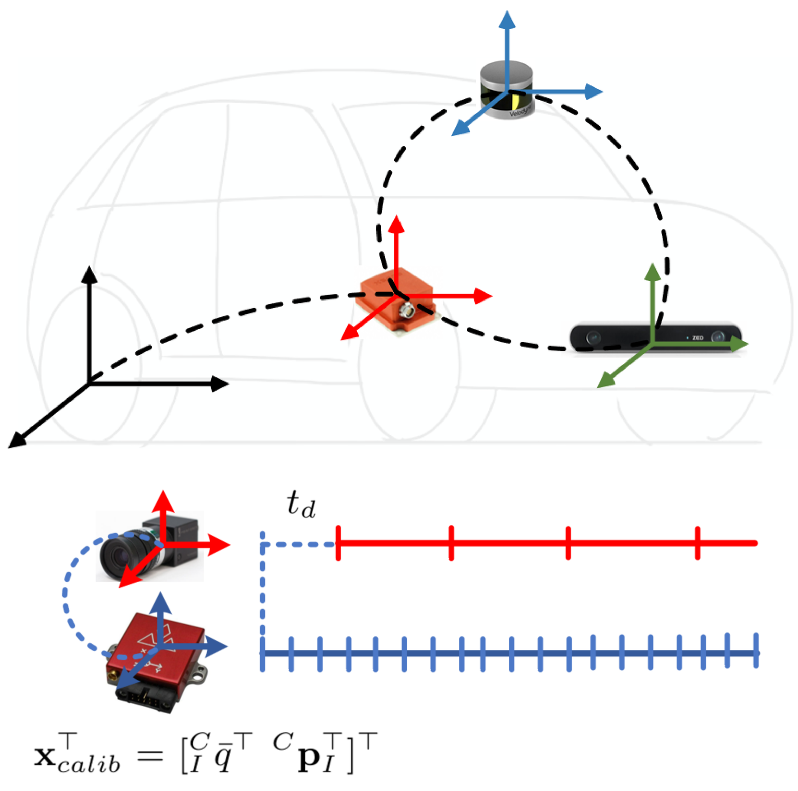
Y. Yang, P. Geneva,
K. Eckenhoff, and G. Huang



RPNG

Motivation

- Aided inertial navigation is one of the most popular 6DOF pose estimation methods.
- Spatial and temporal calibration are vital for fusing exteroceptive measurements with inertial information.
- Degenerate motions may cause some calibration parameters unobservable



Spatial and temporal calibration for aided inertial navigation system.

Contributions

- Both spatial and temporal calibration parameters are observable for any-source aided INS under general motions.
- We identify 4 degenerate motion primitives causing online spatial/temporal calibration to partially fail.
- These identified degenerate motions still hold even when global pose measurements are present.

System Model

- State vector containing IMU, calibration and feature state:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_I^\top & \mathbf{x}_{calib}^\top & t_d & \mathbf{x}_\mathbf{f}^\top \end{bmatrix}^\top$$

- State transition matrix:

$$\begin{bmatrix} \mathbf{\Phi}_{I(k,1)} & \mathbf{0}_{15\times 6} & \mathbf{0}_{15\times 1} & \mathbf{0}_{15\times 3} \\ \mathbf{0}_{6\times 15} & \mathbf{\Phi}_{Calib(k,1)} & \mathbf{0}_{5\times 1} & \mathbf{0}_{6\times 3} \\ \mathbf{0}_{1\times 15} & \mathbf{0}_{1\times 6} & \mathbf{\Phi}_{td(k,1)} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{3\times 15} & \mathbf{0}_{3\times 6} & \mathbf{0}_{3\times 1} & \mathbf{\Phi}_{\mathbf{f}(k,1)} \end{bmatrix}$$

- Observability analysis:

$$\mathbf{M}(\mathbf{x}) = \begin{bmatrix} \mathbf{H}_{I_1}\mathbf{\Phi}_{(1,1)} \\ \mathbf{H}_{I_2}\mathbf{\Phi}_{(2,1)} \\ \vdots \\ \mathbf{H}_{I_k}\mathbf{\Phi}_{(k,1)} \end{bmatrix}$$

N is the unobservable Space If:

$$\mathbf{M}(\mathbf{x})\mathbf{N} = \mathbf{0}$$

Degenerate Motion

- Given random motion, spatial and temporal calibration are observable.
- Pure translation: translation part of spatial calibration is unobservable.
- One-axis rotation: translation part of spatial calibration along the rotation axis is unobservable.
- Constant local angular and linear velocity: time offset is unobservable.
- Constant local angular velocity and global linear acceleration: time offset is unobservable.

Degenerate Summary

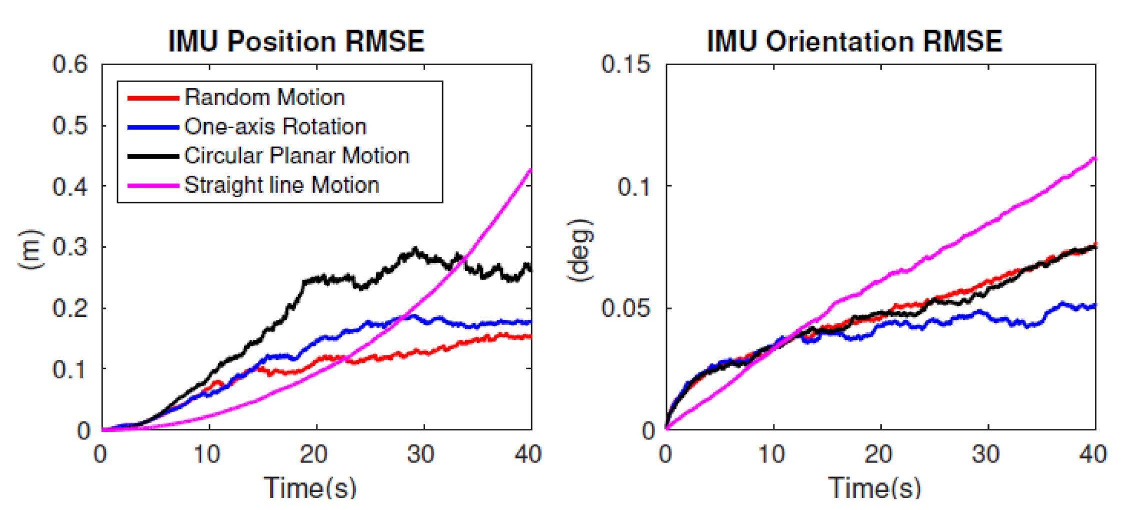
Motion	Unobservable	Observable
No motion	${}^C\mathbf{p}_I$, ${}^C_I\mathbf{R}$ and t_d	–
Pure Translation	${}^C\mathbf{p}_I$	${}^C_I\mathbf{R}$ and t_d
One-axis Rotation	${}^C\mathbf{p}_I$ along rotation axis	${}^C_I\mathbf{R}$ and t_d
Constant ${}^I\boldsymbol{\omega}$ Constant ${}^I\mathbf{v}$	t_d and ${}^C\mathbf{p}_I$ along rotation axis	${}^C_I\mathbf{R}$
Constant ${}^I\boldsymbol{\omega}$ Constant ${}^G\mathbf{a}$	t_d and ${}^C\mathbf{p}_I$ along rotation axis	${}^C_I\mathbf{R}$
One global axis translation Two-axis rotation	–	${}^C_I\mathbf{R}$, ${}^C\mathbf{p}_I$, t_d
Random motion	–	${}^C_I\mathbf{R}$, ${}^C\mathbf{p}_I$, t_d

Simulation Setup

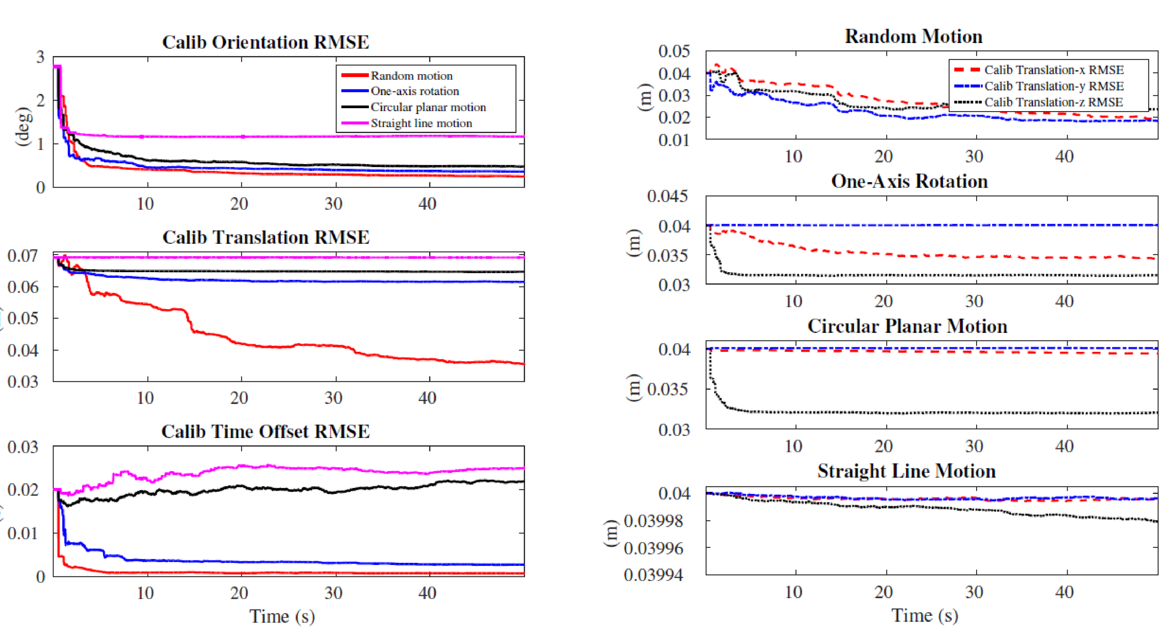
Parameters	Values
True ${}^C_I\bar{q}$	$[0, 0, 0, 1]^\top$
True ${}^C\mathbf{p}_I$	$[0.01, 0.02, 0.02]^\top$ (m)
True t_d	0.04 (sec)
Initial ${}^C_I\bar{q}$	$[0.0099, 0.0198, -0.0099, 0.9997]^\top$
Initial ${}^C\mathbf{p}_I$	$[0.05, 0.06, -0.02]^\top$ (m)
Initial t_d	0.02 (sec)
Initial σ for calib orientation	0.04 (rad)
Initial σ for calib translation	0.05 (m)
Initial σ for calib time offset	0.02 (sec)

Simulations

- Online MSCKF with spatial/temporal calibration with 4 motion primitives

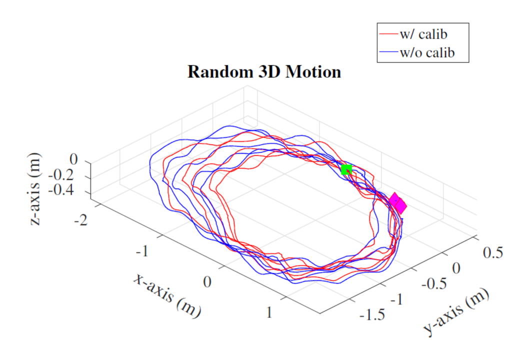


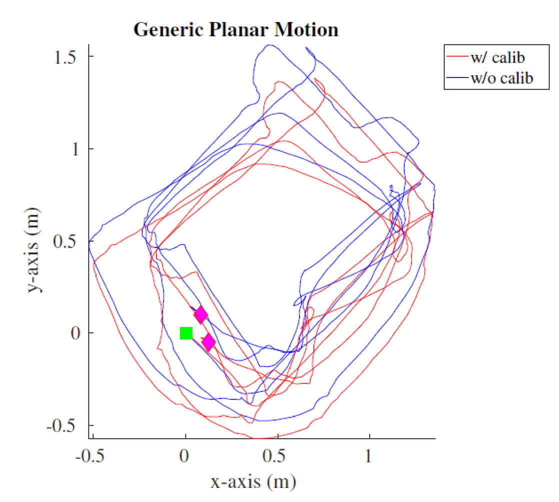
- The convergence for spatial and temporal calibration parameters

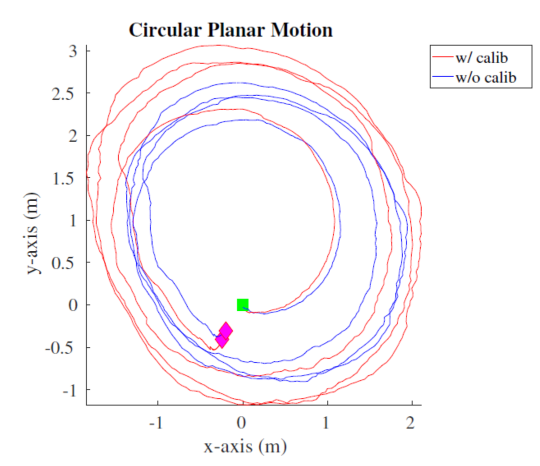


Experiments

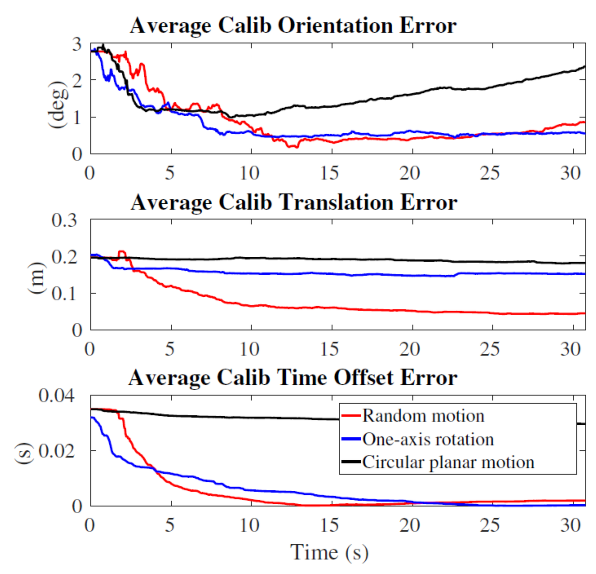
- Real-world experiments with 3 motion models:







- All calibration parameters are observable under random motion.
- Translation unobservable with one-axis rotation motion.
- Time offset and translation unobservable with circular planar motion.



Summary

- **Conclusions**:
  - Performed observability analysis for linearized aided INS with both spatial and temporal calibration and showed that calibration are observable.
  - Identified four non-trivial degenerate motions that might cause online spatial/temporal calibration to fail.
  - Unobservable directions still hold even when global pose measurements are available.
- **Future work:**
  - Extend the current work to multi-sensor calibration with both spatial/temporal calibration.
  - Investigate the case when the time offset is time-varying.